

Horizontal Merger and Acquisition Review based on Consumer Surplus Increment

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Abstract

Based on Cournot model and Dixit differentiated demand model, taking the level of consumer surplus as the welfare standard of anti-monopoly control, and with the help of consumer surplus function, firstly, this paper deduces the marginal cost value of keeping the level of consumer surplus unchanged after the merger of two companies, that is c_n . Then, the paper uses mathematical formulas to prove that the effective conditions of horizontal M&A review is: $c_{ij} < c_n$. At the same time, for companies with high degree of market asymmetry and large scale, more stringent approval rules should be used.

Keywords

Horizontal M&A; Consumer Surplus; Merger Approval.

1. Introduction

Companies M&A includes horizontal M&A, vertical M&A and conglomerate M&A, which are important driving forces of enterprise dynamic development[1]. Generally speaking, operational synergy and financial synergy are the main purposes of any M&A. Almost all open economies have their own merger policies, and with the increase of free trade, the degree of strictness of merger policies is also increasing[2]. From the actual data, there are many examples of companies failing after the M&A[3]. It is worth nothing that a decision-making error in merger approval is often more significant than the error of the merger company itself [4], before submitting the M&A plan for approval, the M&A company should also conduct internal analysis to improve the probability of approval, because once the regulatory agency rejects the M&A approval, regardless of whether the final merger plan is passed or not, the process cost is highly expensive.

In horizontal merger supervision and approval decision-making, industry concentration degree CR_n , HHI index etc, combined with demand elasticity test, to determine whether the merger plan (before) or merger event (after) is enough to cause monopoly attention. The above indicators are based on market share and clear market boundaries. Under the current situation and development trend of free trade, the feasibility of defining market boundaries is facing challenged. In fact, the net growth target of consumer surplus in the approval decision of M&A plan can be controlled, rather than directly using the net growth index of social welfare with the impact of interest contradiction and information asymmetry as the control variable of M&A[5, 6]. Research by Shan [7] shows that China's M&A regulatory practice abides by the economic logic of net social welfare growth, giving consumers more weight on surplus indicators.

In this paper, based on the constraints and mathematical models set in the above research, three asymmetric companies with product differentiation and certain substitutability are set up, and their Dixit differentiated demand functions[8] are constructed respectively. Using Cournot model and Dixit utility function, the consumer surplus in the market equilibrium

before and after merger is calculated. The main contents of this paper are: put forward and prove two propositions, which constitute the conditions of approval validity, that is $c_{ij} < c_n$, which can provide reference for regulatory authorities to examine and approve merger proposals.

2. Asymmetric M&A

2.1. System Description

In an industry oligopoly market, there are three companies A, B and C competing with each other. At the same time, they are negotiating merger plans and submitting them to the regulatory authorities for approval. Let i, j, k denote A, B and C respectively. The marginal cost of each company before merger is c_i, c_j, c_k , and $c_i < c_j < c_k$. The output is divided into three parts, i.e.: q_i, q_j, q_k , the price: p_i, p_j, p_k . The market equilibrium total output is Q_0 .

Assuming that the potential synergy of the above companies remains unchanged for a certain period of time, the change of preference caused by the past negotiation failures can be ignored, and the market meets the following three conditions:

First, the regulatory authorities announce a standard for the approval of M&A and promise to remain unchanged for a period of time;

Second, all the companies involved in the merger know how much synergy they will generate;

Third, if no merger proposal is adopted, then the tri-partite monopoly will continue to exist until the implementation of the next merger proposals.

There are three rules in this paper: the ex-ante approval, the ex-post approval rule and effective approval rules. The ex-ante approval refers to the stage after the merger proposal is submitted and before the actual M&A activities, whether the regulatory authorities vote on the merger proposal to adopt it or not. The ex-post approval rule is based on the actual performance of the merged company within the period of M&A agreed by the merging company and the regulatory authorities to decide whether to stop merger activities. Effective approval rules refer to that the regulatory authorities examine and approve merger proposals on the principle of not reducing the level of consumer surplus, which is an ideal way. In practice, there are two ways to ensure the realization of effective merger, that are the ex-ante approval and the ex-post approval rule. For merger negotiation, the following two rounds of non-cooperative negotiation pattern are adopted:

- (1) If any one of the three companies initiates an invitation for merger, set the company as X;
- (2) Company X invited one of the other two companies to become a negotiating partner, making the company Y;
- (3) Company Y chooses to accept or reject the invitation. If it does, company (X, Y) will enter the negotiation stage, otherwise the company (Y, Z) or (X, Z) enter the negotiation stage. (with (F, E) denotes the companies entering the negotiation)
- (4) Randomly select one of the two companies as the acquirer, that is F, then E company is the target company;
- (5) Company F makes an offer to company E: θ_F^E
- (6) Company E chooses to accept or reject the offer; If E accepts, then E gets θ_F^E , F gets $(\pi_{FE} - \theta_F^E)$. If E refuses the offer, then all the companies will get the equilibrium profit π_{i0} in this period. And restart negotiations in the next period.

2.2. Consumer Surplus before Merger

Let CS_0 denotes the consumer surplus level before M&A. According to Dixit model, the demand functions of the three companies can be described as follows:

$$p_i = \alpha - \beta q_i - \gamma(q_j + q_k) \tag{1}$$

$$p_j = \alpha - \beta q_j - \gamma(q_i + q_k) \tag{2}$$

$$p_k = \alpha - \beta q_k - \gamma(q_i + q_j) \tag{3}$$

where $\alpha > 0, \beta > \gamma > 0$. This paper uses Cournot model to analyze the output competition, so it needs to meet $(3\beta + 2\gamma)c_i > \gamma\alpha, \beta\alpha > (\beta + \gamma)c_k$ [9]. Then the profit function of company A is: $R_i = (p_i - c_i)q_i = (\alpha - c_i - \beta q_i - \gamma \sum_{j \neq i} q_j)q_i$. For the profit function, if the first derivative is obtained, the following equation holds when the profit of company A is the largest:

$$\alpha - c_i - 2\beta q_{i0} - \gamma(q_{j0} + q_{k0}) = 0 \tag{4}$$

Similarly, the conditions for company B and C to maximize profits are as follows:

$$\alpha - c_j - 2\beta q_{j0} - \gamma(q_{i0} + q_{k0}) = 0 \tag{5}$$

$$\alpha - c_k - 2\beta q_{k0} - \gamma(q_{i0} + q_{j0}) = 0 \tag{6}$$

By solving the equations (4)-(6), we can find the Cournot equilibrium output of the market before the merger:

$$Q_0 = q_{i0} + q_{j0} + q_{k0} = \frac{3\alpha - (c_i + c_j + c_k)}{2(\beta + \gamma)} \tag{7}$$

Let $\frac{c_i - c_j}{\gamma - 2\beta} = A, \frac{c_i - c_k}{\gamma - 2\beta} = B$, then $q_{j0} = q_{i0} - A, q_{k0} = q_{i0} - B$, the output of three companies can be calculated:

$$q_{i0} = \frac{\alpha + (A+B)\gamma - c_i}{2(\beta + \gamma)} \tag{8}$$

$$q_{j0} = \frac{\alpha + (B-A)\gamma - c_i - 2A\beta}{2(\beta + \gamma)} \tag{9}$$

$$q_{k0} = \frac{\alpha + (A-B)\gamma - c_i - 2B\beta}{2(\beta + \gamma)} \tag{10}$$

According to Dixit utility function [10]:

$U(Q, x) = x + \alpha \sum_{i=1}^3 q_i - \frac{1}{2}(\beta \sum_{i=1}^3 q_i^2 + 2\gamma \sum_{i=1}^3 \prod_{i \neq j} q_i q_j)$, where x stands for competitive unit pricing, so we can get the consumer surplus function:

$CS = U(Q, x) - \sum_{i=1}^3 q_i p_i = x + \alpha \sum_{i=1}^3 q_i - \frac{1}{2}(\beta \sum_{i=1}^3 q_i^2 + 2\gamma \sum_{i=1}^3 \prod_{i \neq j} q_i q_j) - \sum_{i=1}^3 q_i p_i$, bring the expression of q_{i0}, q_{j0}, q_{k0} into CS function, we get the consumption surplus level before merger:

$$CS_0 = x + \frac{\beta-\gamma}{2} \sum_{i=1}^3 q_{i0}^2 + \frac{\gamma}{2} (\sum_{i=1}^3 q_{i0})^2 \tag{11}$$

2.3. Consumer Surplus Level after M&A

Hypothetically let company A and B merge, we record c_{ij} as the marginal cost of the merged company AB. Internal production does not participate in the competition. In order to facilitate the subsequent derivation, this paper assumes the joint output of the merged company is q_{ij} . c_{k1} is the marginal cost of individual company after merger, The output of individual companies is q_{k1} . After the completion of merger, the profit function of AB is:

$$R_{ij} = 2(p_{ij} - c_{ij})q_{ij} = 2[\alpha - c_{ij} - (\beta + \gamma)q_{ij} - \gamma q_{k1}]q_{ij} \tag{12}$$

Let R_{ij} do the first partial derivative for q_{ij} . When the profit is maximized:

$$\alpha - c_{ij} - 2(\beta + \gamma)q_{ij} - \gamma q_{k1} = 0 \tag{13}$$

Similarly, for the single company, the profit maximization of C is:

$$\alpha - c_k - 2\beta q_{k1} - 2\gamma q_{ij} = 0 \tag{14}$$

Simultaneously solve equations (13) (14):

$$q_{ij} = \frac{2\beta(\alpha - c_{ij}) - \gamma(\alpha - c_{k1})}{4\beta(\beta + \gamma) - 2\gamma^2} \tag{15}$$

$$q_{k1} = \frac{2(\beta + \gamma)(\alpha - c_{k1}) - 2\gamma(\alpha - c_{ij})}{4\beta(\beta + \gamma) - 2\gamma^2} \tag{16}$$

Bring q_{ij} and q_{k1} into the CS function, the consumer surplus level in the market after merger is:

$$CS_1 = x + \frac{\beta-\gamma}{2} (2q_{ij}^2 + q_{k1}^2) + \frac{\gamma}{2} (2q_{ij} + q_{k1})^2 \tag{17}$$

3. Reasonable Conditions of Merger Approval

Proposition 1: when the degree of market asymmetry meets certain conditions, any two companies reach a merger agreement, then there exists the marginal cost of the merger company, c_n , which makes the market consumer surplus remain unchanged.

Proof: let $CS_0 = CS_1$, so the following equation exists:

$$\begin{aligned} & \frac{\beta}{2}(q_{i0} + q_{j0} + q_{k0})^2 + (\gamma - \beta)(q_{i0}q_{j0} + q_{j0}q_{k0} + q_{i0}q_{k0}) \\ &= \frac{\beta - \gamma}{2}(2q_{ij}^2 + q_{k1}^2) + \frac{\gamma}{2}(2q_{ij} + q_{k1})^2 \end{aligned} \tag{18}$$

Take q_{i0}, q_{j0}, q_{k0} to the formula (18), let $(q_{i0}q_{j0} + q_{j0}q_{k0} + q_{i0}q_{k0})$ as G , $c_i + c_j + c_k = c_T$, therefore:

$$CS_0 = x + (\gamma - \beta) \frac{G}{4(\beta + \gamma)^2} + \frac{\beta}{2} \left[\frac{(3\alpha - c_T)^2}{4(\beta + \gamma)^2} \right] \tag{19}$$

Then the right part of formula (18), $\frac{\beta - \gamma}{2}(2q_{ij}^2 + q_{k1}^2) + \frac{\gamma}{2}(2q_{ij} + q_{k1})^2$, whose molecular parts can be arranged as follows:

$$\begin{aligned} & (4\beta^3 + 4\beta^2\gamma - 6\beta\gamma^2)(\alpha - c_n)^2 + 4\gamma^3(\alpha - c_{k1})(\alpha - c_n) \\ & + [(2\beta^2 + 4\beta\gamma + 3\gamma^2)(\beta - \gamma) + 2\beta^2\gamma](\alpha - c_{k1})^2 \end{aligned} \tag{20}$$

Let $\alpha - c_n = T$, $4\beta^3 + 4\beta^2\gamma - 6\beta\gamma^2 = E$, $4\gamma^3(\alpha - c_{k1}) = D$, $[(2\beta^2 + 4\beta\gamma + 3\gamma^2)(\beta - \gamma) + 2\beta^2\gamma](\alpha - c_{k1})^2 = C$. Thus the formula (20) can be written as $ET^2 + DT + C$. Then we can get the level of consumer surplus after the merger, CS_1 . And the further equation can be written as follows:

$$\frac{ET^2 + DT + C}{[4\beta(\beta + \gamma) - 2\gamma^2]^2} = \frac{(\gamma - \beta)G + \frac{\beta}{2}(3\alpha - c_T)^2}{4(\beta + \gamma)^2} \tag{21}$$

Now we note $\frac{(\gamma - \beta)G + \frac{\beta}{2}(3\alpha - c_T)^2}{4(\beta + \gamma)^2} \times [4\beta(\beta + \gamma) - 2\gamma^2]^2$ as M . Then (21) can be written as: $ET^2 + DT + C - M = 0$, You can see $E = 4\beta^3 + 4\beta^2\gamma - 6\beta\gamma^2 > 0$, $D = 4\gamma^3(\alpha - c_{k1}) > 0$. When the degree of market asymmetry before merger meets: $\Delta \geq 0$, where $\Delta = (D^2 - 4E(C - M))$, it can be known that the equation has a solution:

$T_{ij} = \frac{-D \pm \sqrt{D^2 - 4E(C - M)}}{2E}$ (make it positive), finally, $c_{n(ij)} = \alpha - T_{ij}$, similarly, it can be calculated with $c_{n(jk)}$ or $c_{n(ik)}$.

Proposition 2: Only when the marginal cost of merger company meets the condition: $c_{ij} < c_n$, can the level of consumer surplus be improved.

Proof: taking AB merger as an example, the consumer surplus in the market after AB merger is as follows:

$$CS(c_{ij}) = x + \frac{\beta - \gamma}{2} (2q_{ij}^2 + q_{k1}^2) + \frac{\gamma}{2} (2q_{ij} + q_{k1})^2 \tag{22}$$

After sorting:

$$\begin{aligned} CS(c_{ij}) &= \frac{\beta - \gamma}{2} \times \frac{2[2\beta(\alpha - c_{ij}) - \gamma(\alpha - c_{k1})]^2 + [2(\beta + \gamma)(\alpha - c_{k1}) - 2\gamma(\alpha - c_{ij})]^2}{[4\beta(\beta + \gamma) - 2\gamma^2]^2} + x \\ &+ \frac{\gamma}{2} \times \frac{[4\beta(\alpha - c_{ij}) - 2\gamma(\alpha - c_{k1}) + 2(\beta + \gamma)(\alpha - c_{k1}) - 2\gamma(\alpha - c_{ij})]^2}{[4\beta(\beta + \gamma) - 2\gamma^2]^2} \\ &= x + \frac{[(4\beta^2 + 2\gamma^2)(\beta - \gamma) + 2\gamma(2\beta - \gamma)^2](\alpha - c_{ij})^2 + 4\gamma^3(\alpha - c_{k1})(\alpha - c_{ij}) + [\gamma^2 + 2(\beta + \gamma)^2](\beta - \gamma)(\alpha - c_{k1})^2}{[4\beta(\beta + \gamma) - 2\gamma^2]^2} \end{aligned}$$

Therefore, $\frac{dCS}{dc_{ij}} = \frac{-2[(4\beta^2 + 2\gamma^2)(\beta - \gamma) + 2\gamma(2\beta - \gamma)^2](\alpha - c_{ij}) - 4\gamma^3(\alpha - c_{k1})}{[4\beta(\beta + \gamma) - 2\gamma^2]^2}$

As for $\alpha > 0, \beta > \gamma > 0, \beta\alpha > (\beta + \gamma)c_k$, thus $(4\beta^2 + 2\gamma^2)(\beta - \gamma) > 0, 2\gamma(2\beta - \gamma)^2 > 0, \alpha - c_{ij} > 0, \alpha - c_{k1} > 0$, so $\frac{dCS}{dc_{ij}} < 0$. It can be proved in other cases. All in all, the approval criteria for the regulatory authorities to guarantee the increase of consumer surplus level is: $c_{ij} < c_n$.

4. Summary

This paper studies the merger model of three asymmetric companies, and uses mathematical model to deduce the marginal cost critical point of keeping consumer surplus unchanged after merger, that is c_n . At the same time, we use the formula to prove that we should ensure the marginal cost after merger meets: $c_{ij} < c_n$. In order to improve the level of consumer surplus, when the degree of market asymmetry is large, the conflict of interest caused by the merger proposal is more acute. For the approval of a merger proposal, the marginal cost of the merger company is affected by many factors, so the regulatory authorities need to investigate carefully in advance. Strict ex-ante approval rules can reduce the possibility of invalid merger; at the same time, continue to follow up the merger situation after the event. On the one hand, the merger company can stop loss in time, on the other hand, it can ensure that the merger proposal does not damage the consumer surplus level.

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